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The second Born approximation in electron-helium scattering in a Nd-YAG laser field

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Abstract. The dynamics of laser-assisted elastic collisions in helium is studied using the second-order Born approximation. Detailed calculations of the scattering amplitudes are performed by using the Sturmian basis expansion. Differential cross sections for elastic scattering with the net absorption/emission of up to two photons are calculated for collision energies of 5 eV, 10 eV, and 20 eV. We discuss the influence of the low-energy electrons on the differential cross section (DCS) as a function of the scattering angle for selected choices of the laser frequency and the number of photons exchanged between the external field and electron-helium system.

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1 Introduction

The study of atomic systems interacting with intense laser fields has attracted considerable interest in recent years. The observation of multiphoton free-free absorption and emission in elastic electron-atom collisions in the presence of a CO_2 laser [1] represented the first demonstration of a laser-assisted multiphoton process. With the availability of lasers it has become possible to make detailed studies of these differential cross sections, not only for elastic scattering, but also for inelastic collisions (excitation, ionization, ...). The challenge for theory lies in accurately treating each of the electron-target, laser-electron and laser-target interactions. Perturbative treatments may be used if one of these dominates the others. In an early work on free-free scattering, for example, Kroll and Watson [2] treated the laser-electron interaction with higher order terms in the Born series, while the dressing of the target by the field was neglected, to obtain a formula that is valid when the frequency ω of the laser field is much smaller than the kinetic energy of the incident electron. The experimental data concerning large-angle scattering are in reasonable agreement with the Kroll-Watson-type approximations (KWA), which neglect the internal degrees of freedom of the atom. In KWA, the differential cross section for free-free scattering is expressed as the product of the field-free differential cross section evaluated at shifted initial and final electron momenta and a factor that depends on the field and the electron momentum transfer. The derivation of the KWA breaks down at critical geometries where the direction of the electric field is perpendicular to the momentum transfer, but the differential cross sections for these geometries are expected to be very small [2].

Several experiments have been performed, in which the exchange of one or more photons between the electron-atom system and the laser field has been observed [3–8]. Moreover, the laser field introduces new parameters into the description of the collision such as its intensity, its frequency, and its polarization. At present, almost all the free-free experiments have been performed with a CO_2 laser as the radiation field ($\hbar\omega = 0.117$ eV) using helium and argon as atomic-target. For such cases a number of experiments have verified qualitatively the predictions of the KWA at large scattering angles [3]. However, in early experiments on argon and helium targets, at critical geometries, where the laser polarization is almost perpendicular to the momentum transfer, Wallbank and Holmes [4–6] have measured angular distributions several orders of magnitude larger than those predicted by KWA. They suggested that the disagreement could be due to the

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polarization of the target by the field and/or its dressing effects (the effects of the internal degrees of freedom of the atom).

The aim of this work is to generalize our previous treatment of the case of a helium target, for which the prospects of performing experiments are favourable. We will provide a new analysis of our previous work [9,10] in particular at low collision energies where most experiments were performed and the results qualitatively agree with KWA.

The paper is structured as follows. In Section 2 we present the general formation of laser-assisted elastic electron-atom collisions in the case of linear polarization at low collision energies. An account is then given of the techniques we have used to evaluate the scattering amplitudes. Section 2 also contains details of our numerical results as well as their physical interpretation and interest. Unless otherwise stated atomic units (au) are used throughout.

2 Theory, results and discussion

Following our previous work [9,10], we consider a classical monochromatic and single-mode laser field that is spatially homogeneous. This means that it varies little over the atomic range and that the dipole approximation is valid. Working in the Coulomb gauge, we have for the vector potential of a field propagating along the $\hat{\mathbf{z}}$-axis and represented in the collision plane $(\hat{\mathbf{x}} - \hat{\mathbf{y}})$

$$\mathbf{A}(t) = A_0 \left[\hat{\mathbf{x}} \cos(\omega t + \varphi) + \hat{\mathbf{y}} \sin(\omega t + \varphi) \tan\left(\frac{\eta}{2}\right)\right], \quad (1)$$

with the corresponding electric field

$$\mathcal{E}(t) = \mathcal{E}_0 \left[\hat{\mathbf{x}} \sin(\omega t + \varphi) - \hat{\mathbf{y}} \cos(\omega t + \varphi) \tan\left(\frac{\eta}{2}\right)\right], \quad (2)$$

where $\mathcal{E}_0 = \omega A_0/c$, $\mathcal{E}_0$ and ω are the peak electric field strength and the laser angular frequency, respectively. Here η measures the degree of ellipticity of the field and the particular cases of linear polarization ($\eta = 0$) and circular polarization ($\eta = \pi/2$) are easily recovered. Here φ denotes the initial phase of the laser field. We can rewrite the electric laser field in terms of its spherical components by

$$\mathcal{E}(t) = \mathcal{E}_0 \sum_{\nu=\pm 1} i\nu \hat{\varepsilon}_\nu \exp(-i\nu(\omega t + \varphi)), \quad (3)$$

where $\hat{\varepsilon}_\nu = [\hat{\mathbf{x}} + i\nu\hat{\mathbf{y}} \tan(\eta/2)]/2$ is the polarization vector.

The process in which ℓ photons from the laser are exchanged, during the electron-helium elastic collision, can be described by the following equation

$$e^-(\mathbf{k}_0, E_{\mathbf{k}_0}) + \mathrm{He}(1^1\mathrm{S}) + \ell\hbar\omega \longrightarrow \mathrm{He}(1^1\mathrm{S}) + e^-(\mathbf{k}_f, E_{\mathbf{k}_f}), \quad (4)$$

where $\mathbf{k}_0$ and $\mathbf{k}_f$ are respectively the momentum of the incident and scattered electrons in the presence of the laser field. $E_{\mathbf{k}_0} = \mathbf{k}_0^2/2$ and $E_{\mathbf{k}_f} = \mathbf{k}_f^2/2$ are the initial and final projectile kinetic energies. The helium target is initially in the ground state 1^1S. The integer ℓ is the number of photons transferred between the electron-target system and the laser field, where positive values of ℓ correspond to the absorption of photons by the system and negative ones to the stimulated emission photons.

The interaction between the projectile and the laser field is treated exactly and its solution is given by the non-relativistic Volkov wave function [9–11]

$$\chi_k(\mathbf{r}_0, t) = (2\pi)^{-3/2} \exp\{i(\mathbf{k}\cdot\mathbf{r}_0 - E_k t - R_k \sin(\omega t - \gamma_k))\}, \quad (5)$$

where $\mathbf{r}_0$ represents the free electron coordinate, $\mathbf{k}$ denotes the electron wavevector and $E_{\mathbf{k}} = \mathbf{k}^2/2$ is the kinetic energy; one also has $\tan(\gamma_k) = (\mathbf{k}\cdot\hat{\mathbf{y}}/\mathbf{k}\cdot\hat{\mathbf{x}}) \tan(\eta/2)$, $R_k = \boldsymbol{\alpha}_0 \left[(\mathbf{k}\cdot\hat{\mathbf{x}})^2 + (\mathbf{k}\cdot\hat{\mathbf{y}})^2 \tan^2(\eta/2)\right]^{1/2}$. $\boldsymbol{\alpha}_0 = \mathcal{E}_0/\omega^2$ represents the oscillation amplitude of a classical electron in a laser field. Since we are interested in fields which have electric strengths smaller than the atomic unit ($\mathcal{E}_0 \ll 5 \times 10^9$ Vcm^{-1}) and frequencies different from the atomic transition energies, perturbation theory is an appropriate method to solve the laser-target interaction process. If one restricts oneself to the first order, the dressed wavefunctions of the atom are well-known (see [11]) and are given by

$$\phi_n(\mathbf{X}, t) = e^{-i\mathbf{A}\cdot\mathbf{R}} e^{-iE_n t} \Bigg[\psi_n(\mathbf{X}) + \frac{i}{2} \sum_{n'} \left(\frac{M^-_{n'n} e^{i\omega t}}{\omega_{n'n} + \omega} - \frac{M^+_{n'n} e^{-i\omega t}}{\omega_{n'n} - \omega}\right) \psi_{n'}(\mathbf{X})\Bigg], \quad (6)$$

where $\mathbf{X}$ denotes the ensemble of the target electrons' coordinates $(\mathbf{r}_1, \mathbf{r}_2)$, $\psi_n(\mathbf{X})$ is the target state of energy E_n in the absence of the laser field, $\omega_{n'n} = E_{n'} - E_n$ is the Bohr frequency and $M^\pm_{n'n} = \mathcal{E}_0 \langle \psi_{n'} | \hat{\boldsymbol{\epsilon}}_\pm \cdot \mathbf{R} | \psi_n \rangle$ is the dipole coupling matrix elements with $\mathbf{R} = \mathbf{r}_1 + \mathbf{r}_2$.

In equation (6) the summation includes an integration over the continuum states. The factor $e^{-i\mathbf{A}\cdot\mathbf{R}}$ ensures gauge consistency between the Volkov free wave functions (5) and the dressed target wave function (6). The wave function (6) is valid for all frequencies ω except, of course, in the vicinity of a Bohr frequency $\omega_{n'n}$.

Remembering that if we consider collision kinematics, where the incident electron is fast and exchange effects are small, we shall, as a first approximation, carry out a first-Born treatment of the scattering process. The S-matrix elements for elastic scattering, in the direct channel, in the presence of the laser field and in the first-Born approximation is given in atomic units by [9–11]

$$S^{B_1}_{el} = -i \times \int_{-\infty}^{+\infty} dt \langle \chi_{\mathbf{k}_f}(\mathbf{r}_0, t)\phi_0(\mathbf{X}, t) | V_d(\mathbf{r}_0, \mathbf{X}) | \chi_{\mathbf{k}_0}(\mathbf{r}_0, t)\phi_0(\mathbf{X}, t)\rangle \quad (7)$$

where

$$V_d(\mathbf{r}_0, \mathbf{X}) = -\frac{2}{r_0} + \sum_{j=1}^{2} \frac{1}{r_{0j}} \quad (8)$$

is the direct electron-atom interaction potential, with $r_{0j} = |\mathbf{r}_0 - \mathbf{r}_j|$. Here, $\chi_{\mathbf{k}_0}(\mathbf{r}_0, t)$ and $\chi_{\mathbf{k}_f}(\mathbf{r}_0, t)$ denote, respectively, the Volkov wave functions of the free electron projectile before and after scattering in the presence of the laser field and $\phi_0(\mathbf{X}, t)$ is the 'dressed' atomic wave function describing the fundamental and final states. This type of contribution to different scattering processes has been previously computed in various instances [9–17]. By expanding the integrand in a Fourier series and integrating over t, we can rewrite equation (7) in the form

$$S_{el}^{B_1} = i(2\pi)^{-1} \sum_{\ell=-\infty}^{+\infty} \delta(E_{\mathbf{k}_f} - E_{\mathbf{k}_0} - \ell\omega) f_{el}^{B_1,\ell}(\boldsymbol{\Delta}), \quad (9)$$

$f_{el}^{B_1,\ell}(\boldsymbol{\Delta})$ represents the first-Born approximation of the elastic scattering amplitude associated with the exchange of ℓ photons and momentum transfer $\boldsymbol{\Delta} = \mathbf{k}_0 - \mathbf{k}_f$. This can be expressed as

$$f_{el}^{B_1,\ell}(\boldsymbol{\Delta}) = f_{elec}^{B_1,\ell}(\boldsymbol{\Delta}) + f_{atom}^{B_1,\ell}(\boldsymbol{\Delta}). \quad (10)$$

The first term on the right-hand side of equation (10), which we shall call 'electronic', describes the scattering of a Volkov electron by the bare atom. The other term, called 'atomic', occurs as a result of 'dressing' effects of the atomic target. The 'electronic' amplitude $f_{elec}^{B_1,\ell}(\boldsymbol{\Delta})$ is given by

$$f_{elec}^{B_1,\ell}(\boldsymbol{\Delta}) = J_\ell(R_\Delta) f_{el}^{B_1}(\boldsymbol{\Delta}) \quad (11)$$

where $f_{el}^{B_1}(\boldsymbol{\Delta}) = -(2/\Delta^2)\langle\psi_0|\tilde{V}_d(\boldsymbol{\Delta}, \mathbf{X})|\psi_0\rangle$ is the field-free first-Born amplitude for elastic scattering, J_ℓ is an ordinary Bessel function of order ℓ.

For the second term, one has

$$f_{atom}^{B_1,\ell}(\boldsymbol{\Delta}) = \frac{i}{2}\left[J_{\ell+l}(R_\Delta) e^{-i\gamma_\Delta} \sum_n \left(\frac{f_{0n}^{B_1}(\boldsymbol{\Delta}) M_{n0}^-}{\omega_{n0} + \omega} + \frac{M_{0n}^- f_{n0}^{B_1}(\boldsymbol{\Delta})}{\omega_{n0} - \omega} \right) - J_{\ell-l}(R_\Delta) e^{i\gamma_\Delta} \sum_n \left(\frac{f_{0n}^{B_1}(\boldsymbol{\Delta}) M_{n0}^+}{\omega_{n0} - \omega} + \frac{M_{0n}^+ f_{n0}^{B_1}(\boldsymbol{\Delta})}{\omega_{n0} + \omega} \right) \right], \quad (12)$$

where $f_{0n}^{B_1}(\boldsymbol{\Delta}) = -(2/\Delta^2)\langle\psi_0|\tilde{V}_d(\boldsymbol{\Delta}, \mathbf{X})|\psi_n\rangle$ and $f_{n0}^{B_1}(\boldsymbol{\Delta}) = -(2/\Delta^2)\langle\psi_n|\tilde{V}_d(\boldsymbol{\Delta}, \mathbf{X})|\psi_0\rangle$ are the first-Born amplitudes corresponding to the scattering event transitions $0 \to n$ and $n \to 0$ in the absence of the laser field and

$$\tilde{V}_d(\boldsymbol{\Delta}, \mathbf{X}) = \sum_{j=1}^{2} \exp(i\boldsymbol{\Delta} \cdot \mathbf{r}_j) - 2. \quad (13)$$

The sums in equation (12) involve only intermediate P states.

The first-Born differential cross section, which accounts for the 'dressing' effects due to the dipole distortion of the target atom by the laser field, for elastic scattering with the transfer of ℓ photons is given by

$$\left(\frac{d\sigma_{el}^{B_1,\ell}}{d\Omega}\right) = \frac{k_f}{k_0} |f_{el}^{B_1,\ell}(\boldsymbol{\Delta})|^2. \quad (14)$$

In contrast to the case of atomic hydrogen [14], an exact evaluation of the expression in equation (12) is not possible, since no general accurate wavefunctions are known for all excited states of helium. Here, we have used the following set of approximations. For the ground state we have used the wavefunction [15]:

$$\psi_{1^1\mathrm{S}}(\mathbf{r}_1, \mathbf{r}_2) = \varphi_0(\mathbf{r}_1)\varphi_0(\mathbf{r}_2) \quad (15)$$

where the orbital $\varphi_0(\mathbf{r})$ for the singlet 1^1S state is given by

$$\varphi_0(\mathbf{r}) = \frac{1}{\sqrt{4\pi}} \mathbf{R}_0^{\mathrm{He}}(r) = \frac{1}{\sqrt{4\pi}} (Ae^{-\alpha r} + Be^{\beta r}) \quad (16)$$

with $A = 2.60505$, $B = 2.08144$, $\alpha = 1.41$ and $\beta = 2.61$.

In the case of elastic scattering from the ground state, we only have to consider n^1P intermediate states which can be represented by expressions of the form [15]

$$\psi_{n^1\mathrm{P}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}} \Big[\psi_{1s}(Z_i, \mathbf{r}_1)\psi_{n^1\mathrm{P}}(Z_0, \mathbf{r}_2) + \psi_{1s}(Z_i, \mathbf{r}_2)\psi_{n^1\mathrm{P}}(Z_0, \mathbf{r}_1) \Big] \quad (17)$$

where ψ_{1s} and $\psi_{n^1\mathrm{P}}$ are hydrogenic wavefunctions corresponding to $1s$ and n^1P states with effective charges $Z_i = 2$ and $Z_0 = 1$, respectively, and the index n can take both discrete and continuous values. Doubly excited states are not taken into account by this method but their contribution is known to be small [16].

A similar analysis of the elastic electron-atom scattering in the presence of a laser field can be made for the higher-order terms of the Born series. As an example, the second-order contribution to the S-matrix element for electron-atom collisions from the ground state to a final state of energy E_0, in the direct channel and in the presence of a laser field accompanied by the transfer of ℓ photons can be given by

$$S_{el}^{B_2} = -i \int_{-\infty}^{+\infty} dt \times \int_{-\infty}^{+\infty} dt' \langle\chi_{\mathbf{k}_0}(\mathbf{r}_0, t)\phi_0(\mathbf{r}, t)|V_d(\mathbf{r}_0, \mathbf{r}) G_0^{(+)}(\mathbf{r}_0, \mathbf{r}, t; \mathbf{r'}_0, \mathbf{r'}, t') \times V_d(\mathbf{r'}_0, \mathbf{r'})|\chi_{\mathbf{k}_0}(\mathbf{r'}_0, t')\phi_0(\mathbf{r'}, t')\rangle, \quad (18)$$

where $G_0^{(+)}$ is the causal propagator. It should be noted that this term as it stands, is second-order in the electron-atom interaction potential V_d and contains atomic wave functions corrected to first-order in the laser field strength $\mathcal{E}_0$. If one retains a global first-order correction in $\mathcal{E}_0$ for the target "dressed" states, one finds that $S_{el}^{B_2}$ is the sum of two terms which are respectively of zero and first-order in $\mathcal{E}_0$ [10].

The study of second-order corrections to atomic S–P amplitudes shows that these corrections tend to a constant value of order k_0^{-1} as Δ becomes small at small scattering angles and thus are rather unimportant in this angular range. However, this is precisely the angular scattering region in which we are interested for $\mathcal{E}_0 \ll 1$ au, because the first-order amplitude is adequate to provide significant dressing effects. These effects supply a contribution of the order of Δ^{-1} and thus govern the differential cross section, while at larger scattering angles the target dressing becomes less important, and under non-resonant conditions one also can model the atom by a structureless centre of force. Therefore, in the following, we shall neglect the second-order contribution to the S-matrix element for laser-assisted collisions calculated in first-order in $\mathcal{E}_0$. When this approximation is adopted, we may concentrate our discussion on the computation of the dominant term, $S_{el}^{B_2,0}$, which describes the collision of a Volkov electron with the undressed atom. It reduces, after a straightforward time integration, to

$$S_{el}^{B_2,0} = -\frac{i}{2\pi^3}\sum_{\ell=-\infty}^{\ell=+\infty}\delta(E_{\mathbf{k}_f}-E_{\mathbf{k}_0}-\ell\omega) \times \sum_{\ell'=0}^{\ell'=+\infty}\sum_n \int d\mathbf{q}\frac{J_{\ell-\ell'}(\boldsymbol{\Delta}_0\cdot\boldsymbol{\alpha}_0)J_{\ell'}(\boldsymbol{\Delta}_0\cdot\boldsymbol{\alpha}_0)}{\Delta_0^4} \times\frac{\langle\psi_0|\tilde{V}_d(\boldsymbol{\Delta}_0,\mathbf{r})|\psi_n\rangle\langle\psi_n|\tilde{V}_d(\boldsymbol{\Delta}_0,\mathbf{r})|\psi_0\rangle}{E_{\mathbf{q}}-E_{\mathbf{k}_0}+\omega_{n,0}-\ell'\omega\pm i\epsilon}, \tag{19}$$

with $\tilde{V}_d(\boldsymbol{\Delta}_0,\mathbf{r}) = \exp(i\boldsymbol{\Delta}_0\cdot\mathbf{r})-1$ and $\boldsymbol{\Delta}_0 = \mathbf{k}_0-\mathbf{q}$. The sum over n runs over the complete set of hydrogenic states and one has to integrate over the virtual projectile states $\chi_{\mathbf{q}}(\mathbf{r}_0,t)$ which represent the wavevector of the free electron at the time of the collision.

Thus, it turns out that the lowest-order component $S_{el}^{B_2,0}$ evaluated at the shifted momenta $\boldsymbol{\Delta}_0$ and using the soft-photon approximation, see for instance reference [13], can be expressed in terms of a simpler second-Born amplitude, as

$$S_{el}^{B_2,0} = -(2\pi)^{-1}i\sum_{\ell=-\infty}^{\ell=+\infty}\delta(E_{\mathbf{k}_f}-E_{\mathbf{k}_0}-\ell\omega)f_{el}^{B_2,\ell,0}(\boldsymbol{\Delta}), \tag{20}$$

with

$$f_{el}^{B_2,\ell,0}(\boldsymbol{\Delta}) = -\frac{J_\ell(R_\Delta)}{\pi^2}\int_0^{+\infty}q^2dqd\Omega_q \times\frac{\langle\psi_0|\tilde{V}_d(\boldsymbol{\Delta}_0,\mathbf{r})G_c(\Omega)\tilde{V}_d(\boldsymbol{\Delta}_0,\mathbf{r})|\psi_0\rangle}{\Delta_0^4} \tag{21}$$

and the electron-atom interaction amplitude with the transfer of ℓ photons may be written, in the second-Born approximation, as

$$f_{el}^{\ell}(\boldsymbol{\Delta}) = f_{el}^{B_1,\ell}(\boldsymbol{\Delta}) + f_{el}^{B_2,\ell,0}(\boldsymbol{\Delta}), \tag{22}$$

where the first-order term $f_{el}^{B_1,\ell}(\boldsymbol{\Delta})$ is given by equation (10) and $G_c(\Omega) = \sum_n |\psi_n\rangle\langle\psi_n|/(\Omega-E_n)$ is the Coulomb Green's function with argument $\Omega = E_{\mathbf{k}_0}-E_{\mathbf{q}}+\ell\omega+E_0$.

In Equation (22) and according to the expansion (21), we may write the second-order $f_{el}^{B_2,\ell,0}(\boldsymbol{\Delta})$ in the form

$$f_{el}^{B_2,\ell,0}(\boldsymbol{\Delta}) = J_\ell(R_\Delta)f_{el}^{B_2}(\boldsymbol{\Delta}) \tag{23}$$

with

$$f_{el}^{B_2}(\boldsymbol{\Delta}) = -\frac{1}{\pi^2}\int_0^{+\infty}q^2dqd\Omega_q \times\frac{\langle\psi_0|\tilde{V}_d(\boldsymbol{\Delta}_0,\mathbf{r})G_c(\Omega)\tilde{V}_d(\boldsymbol{\Delta}_0,\mathbf{r})|\psi_0\rangle}{\Delta_0^4} \tag{24}$$

which is the field-free second-Born elastic amplitude evaluated at the shifted momenta $\boldsymbol{\Delta}_0$.

The first and second-Born elastic amplitudes corresponding, respectively, to the first-order and second-order contribution to the S-matrix element for the laser-assisted elastic scattering process, have been computed exactly without further approximation with the help of a Sturmian approach, similar to the one described in references [10,14]. This constitutes an important advantage in the present context as compared to earlier computations relying on the closure approximation [14,15].

The main problem in evaluating the scattering amplitudes corresponding to the first and the second-order contributions to the S-matrix element for laser-assisted elastic scattering, consists of

(i) performing the summation over the intermediate states. In order to calculate exactly the corresponding radial amplitudes without further approximation, we have used two different techniques based on the Sturmian approach similar to the ones described in our previous work [11,16,21]. This approach allows us to take into account exactly the bound-continuum-state contributions, which are of crucial importance for electron impact scattering at intermediate energies. These methods of computation constitute an important advantage in the present context as compared to earlier ones relying on the closure approximation [15,20];

(ii) the presence of the intermediate wavevector $\mathbf{q}$ in the argument of the Bessel function. Indeed, the integral, (in expression (24)), over the virtual projectile states $\chi_{\mathbf{q}}(\mathbf{r}_0,t)$ with wave vector $\mathbf{q}$ is prohibitively difficult to calculate. It is actually zero at some values of the incident electron energies and accordingly for some values of the scattered electron energies. Each of these possible intermediate transitions will be characterized by a resonance behaviour, i.e. the denominator of the matrix elements entering the exact formula (Eq. (24)) is close to zero. We shall overcome this difficulty by determining the exact upper boundary of the integral (24) over the virtual projectile [9].

In elementary atomic processes identical particles are expected on physical grounds to respond differently to a strong external driving field, the effects due to the particles identity (exchange effects) must become less significant. Basically, the different response to the external perturbation to some extent makes the particles distinguishable. It is well-known from field-free electron atom collision theory that exchange effects lose their importance when the velocity of the incoming electron is considerably larger than that of the atomic electrons. In this case, two identical particles are in quite different physical states. However, in electron-atom collisions, where free and bound electrons are present, a strong driving field should affect in a different way the dynamics of the various electrons, and thus a reduction of exchange scattering amplitude should take place.

The contribution for laser-assisted elastic collisions to the S-matrix of exchange scattering leads to some conceptual difficulties but would not significantly alter the results of the present discussion. We have considered in the present paper only the leading term of $g_{el}^{B_1,\ell}$, the exchange amplitude for electron-atom collisions with the transfer of ℓ photons used in reference [18]. It is known [9,10,18] that the exchange effects in collisions are important at low relative velocities, while the FBA is an essentially high-energy approximation. Thus, the second-Born differential cross section corresponding to the elastic scattering process, with the transfer of ℓ photons, is given by

$$\left(\frac{d\sigma_{el}^{\ell}}{d\Omega}\right) = \frac{k_f}{k_0}|f_{el}^{\ell} - g_{el}^{B_1,\ell}|^2, \tag{25}$$

which does not depend on the initial phase φ of the laser field due to the inability of the collision time to be defined and as a result of the approximation of the projectile wave packet by a mono energetic beam of infinite duration [19].

The present semiperturbative method with the Sturmian basis expansion takes into account the target atom distortion induced by the presence of a laser field. The validity of our treatment is based on the fact that the laser-helium target interaction is non resonant. This condition is more stringent if the laser frequency is comparable to any characteristic atom transition frequency. We note that the elastic scattering process can be considered as non resonant if for a given frequency, the intensity does not exceed a certain limit [15]. Such a condition will be respected by our choice of the Nd-YAG laser frequency $\omega = 1.17$ eV and $\mathcal{E}_0 = 2\times 10^7$ Vcm^{-1}.

We are interested in demonstrating the effects of the incident electron energies in the elastic collision in the presence of a laser field. In helium, there have been comparatively fewer attempts to address the role of the dressing of the atomic states by the strong laser field as the computation is much more complex. This is unfortunate since helium would lend itself more easily than hydrogen to experimental verification. Note however that, though simplified, the model contains all the ingredients needed for the discussion of the physics of such processes. Our results are interpreted by considering the first and second Born differential cross sections, for a fixed electric field strength and a fixed laser photon energy. We have examined our treatment in SBA and FBA as a function of the scattering angles and they give similar results beyond 30 eV for the incoming electron energies.

The results presented in this paper are obtained for a geometry in which the polarization vector of the field $\mathcal{E}_0$ is parallel to the direction of incoming electron wavevector $\mathbf{k}_0$, where free-free differential cross sections are maximum at a particular laser intensity and incident electron energy [7] and where the laser-assisted differential cross section only depends on the orientation of the polarization unit vector $\hat{\varepsilon}_\nu$ [19]. We compare our results in SBA with those obtained in FBA and with the values obtained by using the Kroll-Watson approximation (KWA), where the differential cross sections for the exchange of photons are related to the field-free differential cross section $(d\sigma/d\Omega)$ through

$$\left(\frac{d\sigma^{\ell}}{d\Omega}\right) = \frac{k_f}{k_0}J_\ell^2(R_\Delta)\left(\frac{d\sigma}{d\Omega}\right). \tag{26}$$

In the set of Figures 1–3, we present the differential cross sections for laser-assisted elastic scattering with the net exchange of up to two photons ($\ell = 0, \pm1, \pm2$) when the electron is incident along the laser polarization axis at collision energies of 5 eV, 10 eV and 20 eV. As has already been noted by several authors [16,18–20], the dressing of the electron-target effects are seen to be dominant in the forward direction. This is due to the presence in the 'atomic' term in FBA of the S–P transition amplitude which behaves as Δ^{-1} for small transfer momentum Δ. Moreover, we noticed a destructive interference, in FBA, between the 'electronic' and 'atomic' amplitudes. The presence of such interference is a general feature of $\ell = 0 \longrightarrow \ell = 1$ transitions in the case of inverse bremsstrahlung ($\ell > 0$). This minimum appears at angles for which the first Born differential cross vanishes, i.e. when $f_{elec}^{B_1,\ell}(\boldsymbol{\Delta}) + f_{atom}^{B_1,\ell}(\boldsymbol{\Delta}) = 0$. In contrast, for the cases when the zero-order term of the SBA to the elastic scattering amplitude is taken into account, we do not obtain a deep minimum, because the presence of an additional term forbids the occurrence of complete destructive interference. The inclusion of higher order terms is one typical signature of the dressing of the electron-target system in the differential cross section and clearly shows the effect of internal structure of the atomic target when the energies of the primary electron are low. These are the conditions for which the target distortion induced by the laser field should be taken into account more fully.

For scattering without any net exchange of photons, the differences between the FBA and KWA results are too small to be seen on the scale of Figures 1–3. The earlier work provoked several theoretical investigations on the inherent characteristics of KWA. Geltman [22] and Rabadan [23] have both shown that neglecting the laser-atom interaction has too small an effect under the conditions of the experiments to explain the discrepancy with the KWA. The first-Born approximation results show a completely different behaviour for the cross sections, such that the laser-atom interaction effects are important over a small range of scattering conditions. That's not the case in SBA where the dressing is important over a wide range of incident scattering angles for a given incident energy smaller than 30 eV. We note that, in Figures 1, 2 and 3, when the incident electron energy decreases, the differential cross section is larger in SBA than the first term of the Born series, which shows that the results are sensitive to the higher terms of the Born series for low incident energies. Indeed, one observes strong modifications of the cross sections, as compared with the results of

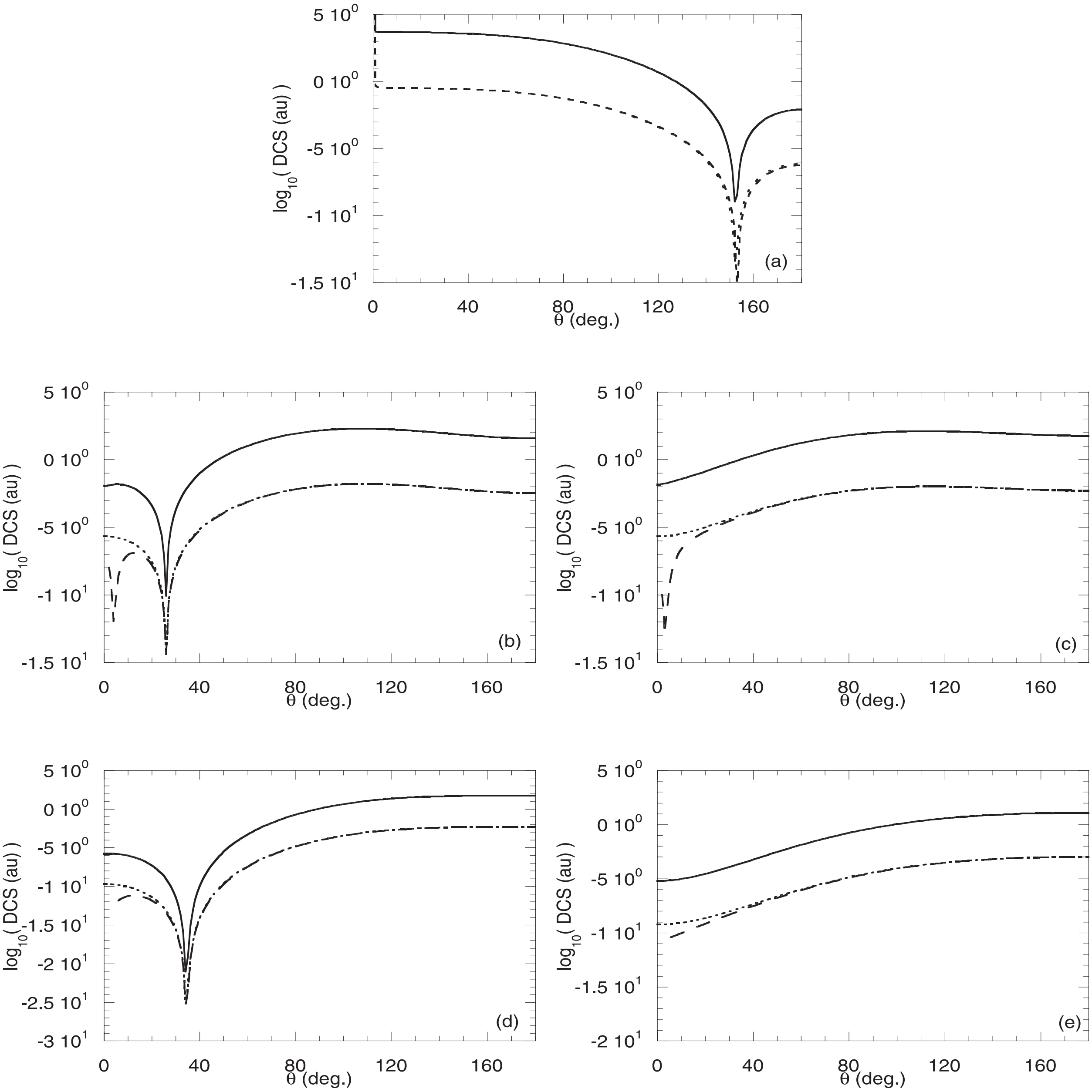

Fig. 1. Variation of $\log_{10}(DCS)$ for elastic scattering with the transfer of ℓ photons $\big[$(a) $\ell = 0$, (b) $\ell = 1$, (c) $\ell = -1$, (d) $\ell = 2$ and (e) $\ell = -2\big]$ as a function of scattering angle (θ). The incident electron energy is 5 eV, the laser frequency is 1.17 eV and the electric field strength is 10^8 Vcm^{-1}. Solid lines: second-Born approximation. Dashed lines: first-Born approximation. Dotted lines: results obtained by neglecting the dressing of the target.

calculations in first-Born approximation and KWA cross sections, in particular at small scattering angles and for low-energy electrons. For a given intensity of the laser field, the minima resulting from the condition $R_\Delta = 0$ move to larger scattering angles as the collision energy increases (the argument of the Bessel function becomes zero if $\ell\omega = E_{k_0} \tan^2\theta$). R_Δ is purely kinematic in origin and is not accessible below $\simeq$4 eV. The position of these minima depend of the intensity and frequency of the laser field, and the kinematic considerations (the choice of the scattering geometry). The absolute magnitude of the differential cross section increases when the incident electron energy decreases and/or $|\ell|$ increases. The situation is different at $R_\Delta = 0$.

The differences between the SBA and KWA are much larger than the differences between the FBA and KWA. Indeed, between FBA and KWA they are almost equal zero. Hence, for low incident energies, the FBA can't show clearly the effect of the laser on the internal structure of the atom in the case where there is no net exchange of photons ($\ell = 0$). Therefore, for low incident energies, the higher order Born-Series are required to display such an

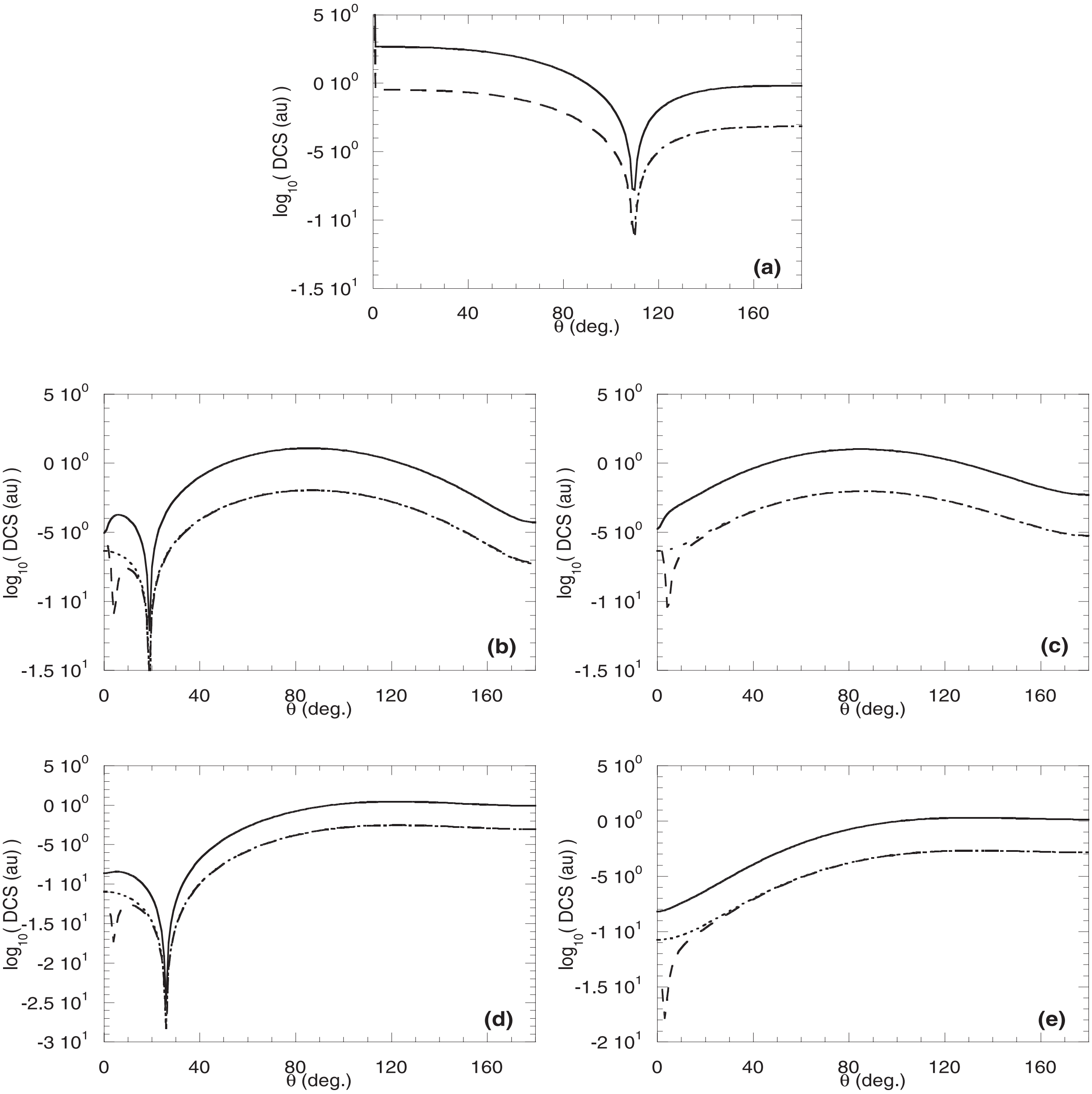

Fig. 2. As in Figure 1, but with an incident electron energy of 10 eV.

effect and thus to interpret correctly the scattering Differential Cross Sections.

The SBA results are generally larger than the corresponding FBA Differential Cross Sections for the different incoming electron energies and at any scattering angle except when the cross section vanishes. For a given energy and a fixed ℓ, the difference between SBA and FBA remains constant for any scattering angle. The results presented in the set of Figures 1–3 illustrate the second-term of Born series effects for incident electron energies in the range 5 to 20 eV: these effects decrease with the incoming electron energies.

For the net exchange of one or two photons, the differences between the FBA and the SBA results are still very important at small scattering angles and are otherwise constant for a given incident energy and a fixed ℓ. This is due to the presence in the 'atomic' term in FBA of S–P transition amplitudes which behave as Δ^{-1} for a small transfer momentum $\boldsymbol{\Delta}$. Moreover, we notice a destructive interference, in FBA, between the 'electronic' and 'atomic' amplitudes. The presence of such interference is a general feature of $1s \rightarrow ns$ transitions in the case of inverse bremsstrahlung ($\ell > 0$). This minimum also appears at angles for which the first Born cross section

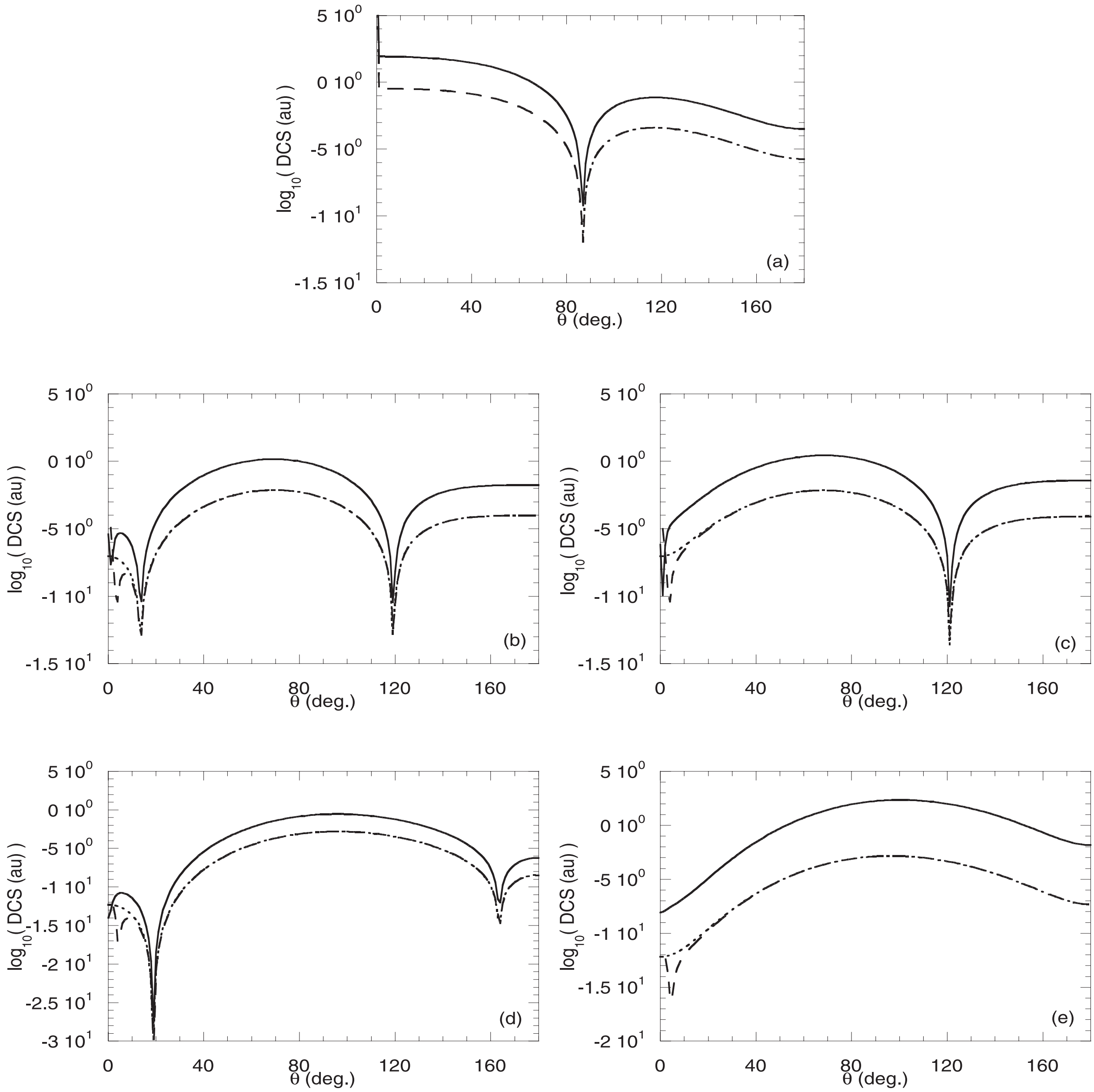

Fig. 3. As in Figure 1, but with an incident electron energy of 20 eV.

vanishes, i.e. when $f_{elec}^{B_1,\ell}(\boldsymbol{\Delta}) + f_{atom}^{B_1,\ell}(\boldsymbol{\Delta}) = 0$. In contrast, for the cases when the zero-order term of the SBA to the elastic scattering is taken into account, we do not obtain a deep minimum, because the presence of an additional term forbids the occurrence of a complete destructive interference. This interference appears clearly at 5 eV. There also appears to be some differences between the differential cross sections due to absorption and also due to emission, in particular, in the forward direction. This difference comes from the presence of a kind of minimum resulting from the condition $R_\Delta = 0$ (the argument of the Bessel function becomes zero if $\ell\omega = \mathcal{E}_0 \tan^2\theta$ in the absorption case. This condition cannot be fulfilled in the emission case. We notice that, in Figures 1, 2 and 3, when the incident electron energy decreases, the differential cross section is larger in SBA than the first term of the Born series for any angles except at angles such that the argument $R_\Delta = 0$ of the Bessel functions actually vanishes. We notice that this zero exists only in the case when $\mathcal{E}_0$ parallel to $\mathbf{k}_0$ and the localization in θ is given by the equation $k_0 - k_f \cos\theta = 0$, where θ is the scattering angle. For the other geometries, this condition cannot be fulfilled. This behaviour shows that the results are sensitive to the higher order terms of the Born series. We show for comparison, in Figures 1, 2 and 3, the second Born differential cross section, which does not have the oscillating structure present in the small-energy region relative to the incident electron.

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